

# Sutton CE (VC) Primary School



# Calculation Policy

This policy has been largely adapted from the White Rose Maths Hub Calculation Policy with further material added. It is a working document and will be revised and amended as necessary.

Approved by Staff in: July 2020

Approved by the Governing Body in: July 2020

## Introduction

This policy aims to ensure that all pupils are supported to:

- Understand the key principles of mathematics, enabling them to become fluent in basic mental and written methods. This will require quick and accurate recall of number bonds and multiplication facts; and sound understanding of place value. With these principles firmly in place, pupils will feel comfortable to test new ideas for themselves and will develop the flexibility necessary to apply their knowledge and work through any problem.
- Apply their knowledge of mathematics to understand and solve problems, including realistic “real world” problems that they may not be familiar with. This will require pupils to identify and understand patterns and relationships; understand information that has been provided with a problem, together with which important information is missing; and to communicate their ideas.
- Use mathematical reasoning, exploring ideas and following a line of enquiry to present reasonable arguments and mathematical proof. This will require pupils to break problems into simpler steps; decide what information to gather or collect for their mathematical solution; and develop persistence in solving problems.

Each mathematical operation is presented in a clear sequence that shows the learning progression from Foundation Stage to Key Stage 2. Many mathematical ideas and methods are linked, so pupils are always encouraged to apply the knowledge that they already possess to new areas of learning.

This document should be used in school to ensure that there is consistency of approaches and complete coverage of the curriculum, and at home to illustrate the important techniques that will be used throughout the children’s mathematical learning journey.

There is strong evidence that children who practise and explore maths at home are more confident and able to apply their thinking to new problems and grasp new concepts more readily. Discussion of maths at home allows the children to consolidate what they learn through repetition and demonstration. The role of parents and carers in the mathematical journey is therefore vitally important.

While the new curriculum allows flexibility in the formal written methods used, there is an expectation that children will be able to use - and will be tested on - specific methods for

calculating with multi-digit numbers. These methods are referred to within this Calculation Policy.

Throughout this document, we show the stages of mathematical learning. These are the steps that the children go through to achieve understanding across the whole curriculum and do not necessarily relate to any particular year. For example, your child might progress through stages 3, 4 and 5 in one year.

## **Organisation**

The policy is organised according to the age stage expectations as set out in the National Curriculum 2014, however it is vital that pupils are taught according to the stage that they are currently working at, moving onto the next level as soon as they are ready, or working at a lower stage until they are secure enough to move on.

Children will use mental methods as their first port of call when appropriate, but for calculations that they cannot do in their heads, they will need to use an efficient written method accurately and with confidence. The policy outlines a range of mental calculation strategies, including the use of jottings, vocabulary to be developed and the key number facts that children will need to know

It is important that any type of calculation is given a real life context or problem solving approach to help build children's understanding of the purpose of calculation, and to help them recognise when to use certain operations and methods when faced with problems.

Children will have Number Talk sessions which support them to explore the most effective method for calculating and share their current method, alongside trying new methods that are accessible to them.

Children need to be taught and encouraged to use the following processes in deciding what approach to take to a calculation and to ensure they select the most appropriate method for the numbers involved:

Is it possible to do the calculation using mental strategies?

Can the calculation be done efficiently using mental strategies supported by jottings?

Are the numbers sufficiently complex to require a formal written calculation method?

## **Fluency in Number**

Developing children's fluency in number facts is considered paramount in order for them to calculate.

## **Inclusion**

Sutton CE (VC) Primary School aims to be responsive to all aspects of diversity and to increase the learning and participation of all children within the school and its locality.

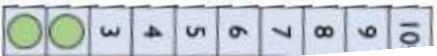
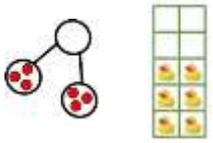
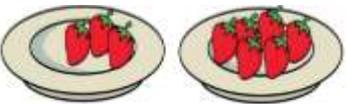
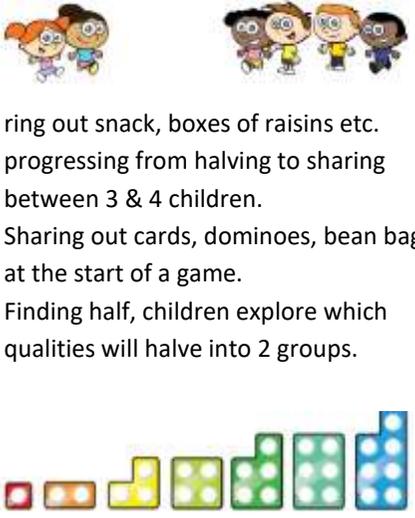
This inclusive culture is reflected in all policies and practices. We ensure that classroom and extra-curricular activities encourage the participation of all children, drawing on their knowledge and experience outside school. Teaching and support are integrated together, enabling all children to overcome barriers to learning and participate fully the life of Sutton School.

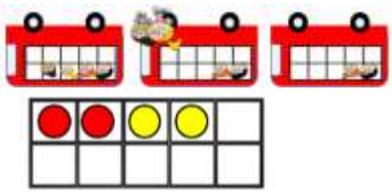
Equality and inclusion will be achieved through analysis and assessment of children's needs, by monitoring the quality of teaching and the standards of children's achievements and by setting targets for improvement. Learning for all children is given equal priority and available resources are used to maximum effect.

Children with Special Educational Needs and disabilities will be given support to access the curriculum at an appropriate level to enable them to reach their full potential.

All children, including those who have been identified as able, gifted and talented, will be given opportunities within lessons and through extra-curricular activities to use and develop their gifts and talents. These opportunities will be provided in accordance with the Gifted and Talented policy.

## Calculation Guidelines for the Early Years Foundation Stage

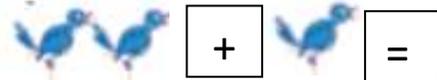
ADDITION	SUBTRACTION	MULTIPLICATION	DIVISION
<b>Children begin to record in the context of play or practical activities and problems.</b>			
<p><b>To add two single-digit numbers and count on to find the answer using quantities and objects.</b></p> <p>Children will initially use real objects to see that the quantity of a group will increase by adding more objects.</p> <p>Activities might include:</p> <ul style="list-style-type: none"> <li>Children roll a 1-3 dice and add that number of bricks or cubes to their towers</li> </ul>  <ul style="list-style-type: none"> <li>Number track race games, rolling a 1-3 dice and move along the track.</li> </ul>  <p>Children to be encouraged to represent, <b>'First, then and now'</b> stories using fingers, tens frames, number tracks and numicon.</p> <ul style="list-style-type: none"> <li>Show me 5 fingers, show me 1 more.</li> </ul>  <ul style="list-style-type: none"> <li>Make a record in pictures, words or symbols of addition activities already</li> </ul>	<p><b>To subtract two single-digit numbers and count back to find the answer using quantities and objects.</b></p> <p>Children will initially use real objects to see that the quantity of a group will change by taking away objects.</p> <p>Activities might include:</p> <ul style="list-style-type: none"> <li>Re-enact favourite rhymes, e.g. 10 green bottles/5 current buns.</li> </ul>  <ul style="list-style-type: none"> <li>Pass it on, chn roll the dice and give away that number of counters to another player</li> </ul>  <ul style="list-style-type: none"> <li>Take a number of objects away from a known group (that are hidden) how many are left?</li> <li>Construct number sentences to go with practical activities</li> </ul>  <ul style="list-style-type: none"> <li>Relate subtraction to taking away and counting how many objects are left.</li> </ul> 	<p><b>To solve problems, including doubling.</b></p> <p>Children will learn that double means twice as many.</p> <p>Activities might include:</p> <ul style="list-style-type: none"> <li>Play snap or matching pairs games</li> <li>Make towers that are double the height or double the length.</li> <li>Hide and seek with numicon, children finding the same to double them.</li> </ul>  <ul style="list-style-type: none"> <li>Play doubles, chn roll 2 dice, if they roll a double they score a point</li> </ul>  <ul style="list-style-type: none"> <li>Children explore other ways of finding doubles</li> </ul>  <ul style="list-style-type: none"> <li>Chn count/chant in twos; fives; tens</li> <li>Children find 2's, 5's &amp; 10's in nature.</li> </ul>	<p><b>To solve problems, including halving and sharing.</b></p> <p>Activities might include:</p> <ul style="list-style-type: none"> <li>Halve quantities by sharing into 2 equal groups.</li> </ul>  <ul style="list-style-type: none"> <li>Organise children into teams</li> <li>Sharing out snack, boxes of raisins etc. progressing from halving to sharing between 3 &amp; 4 children.</li> <li>Sharing out cards, dominoes, bean bags etc. at the start of a game.</li> <li>Finding half, children explore which quantities will halve into 2 groups.</li> </ul>  <ul style="list-style-type: none"> <li>Begin to explore odds and evens</li> </ul> <p><b>Related Vocabulary:</b> How many times? How many are left/left over?</p>

<p>carried out.</p> 	<p>nd one less to ten.</p> <ul style="list-style-type: none"> <li>Counting backwards along a number line</li> </ul> <p>8 - 3 = 5</p> 	 <p>Stories: This is the story of Alison Hubble by Allan Ahlberg Double Trouble – Nrich Number blocks ser2 ep9 – Double trouble</p>	<p>Group / Share out / Half, halve Count in twos, tens What could we try next? How did you work it out? Stories: The Doorbell Rang – Pat Hutchins Bean Thirteen _ Matthew McElligott Maths Story Time - Enrich</p>
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**EYFS + Addition +**

**Vocabulary:** add, more, and, make, sum, total, altogether, score, double, one more, two more, ten more..., how many more to make...?, how many more is... than...?

**MENTAL STRATEGIES:** - Develop a mental image of the number system. - Understand the value of a number - Counting forwards and backwards - Recall of number bonds to 10

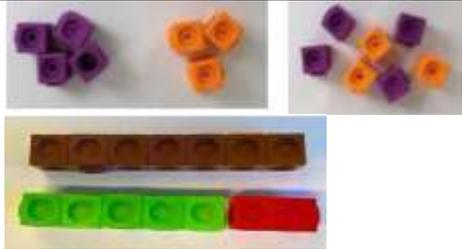
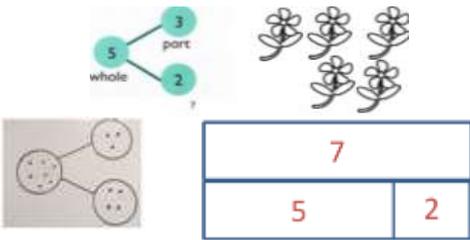
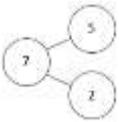
<b>Objective &amp; Strategy</b>	<b>Concrete</b>	<b>Pictorial</b>	<b>Abstract</b>
<p>Using a range of practical resources and real life contexts, pupils develop their understanding of addition through counting activities</p>	<p>How many dinosaurs are there?</p>  <p>What about if I give you two more? How many are there now?</p> 		
<p>Children are introduced to the addition symbol</p>	<p>There are 2 birds. Another bird flies in. How many are there altogether?</p> 		<p align="center"><b>2 + 1 = 3</b></p>
<p>Store the larger number mentally and use fingers to count on</p>	<p>Count on from the larger number. (5 in your head) 'six, seven, eight' using their fingers.</p> 		<p align="center"><b>3 + 5 = 8</b></p>

<p>Children represent an addition number sentence in picture form and are able to solve simple addition number sentences and begin to explain their reasoning</p>			$5 + 2 = 7$
<p>Early number tracks will help children develop their understanding of addition</p>			

**Stage One + Addition +**

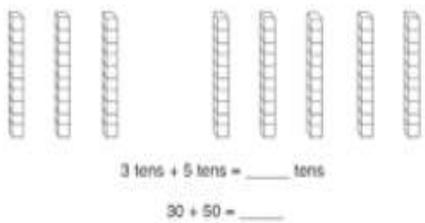
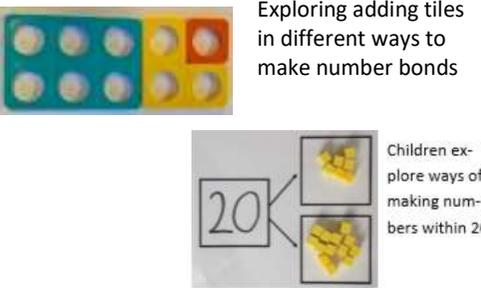
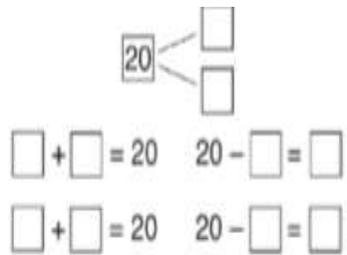
**VOCABULARY:** number bonds, add, more, plus, make, sum, total, altogether, inverse double, near double, equals, is the same as (including equals sign), score, one more, two more... ten more, how many more to make...?, how many more is... than...?, how much more is...?

**MENTAL STRATEGIES:** Children should experience regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10. Children should memorise and reason with number bonds for numbers to 20, experiencing the = sign in different positions. They should see addition and subtraction as related operations. E.g.  $7 + 3 = 10$  is related to  $10 - 3 = 7$ . Use bundles of straws and Dienes to model partitioning teen numbers into tens and ones and develop understanding of place value. Children have opportunities to explore partitioning numbers in different ways. e.g.  $7 = 6 + 1$ ,  $7 = 5 + 2$ , Children should begin to understand addition as combining groups and counting on.

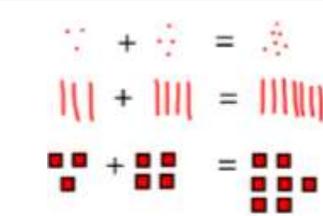
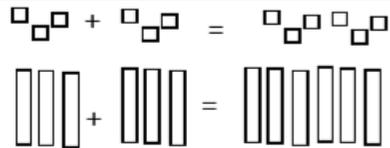
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Combining two parts to make a whole: part- whole model</p>	 <p>Use part whole model. Use objects to add two numbers together as a group or in a bar.</p>	 <p>Use pictures to add two numbers together as a group or in a bar.</p>	 <p style="text-align: right;"><math>2 + 3 = 5</math></p> <p style="text-align: right;"><math>7 = 5 + 2</math></p> <p>Use the part-part whole diagram as shown above to move into the abstract. NB: ensure children understand that ‘=’ means ‘the same as’.</p>



and counting on. They should use Dienes to model partitioning into tens and ones and learn to partition numbers in different ways e.g.  $23 = 20 + 3 = 10 + 13$ .

Objective & Strategy	Concrete	Pictorial	Abstract
<b>Adding multiples of ten</b>	 <p>Model using dienes and bead strings</p>	 <p>3 tens + 5 tens = ___ tens 30 + 50 = ___</p>	$50 = 30 + 20$ $20 + 30 = 50$ $70 = 50 + 20$ $40 + \square = 60$
<b>Use known number facts</b>  <b>If I know ...</b> <b>Then I know.....</b>	 <p>Exploring adding tiles in different ways to make number bonds</p> <p>Children explore ways of making numbers within 20</p>	 <p> <math>\square + \square = 20</math>    <math>20 - \square = \square</math>  <math>\square + \square = 20</math>    <math>20 - \square = \square</math> </p>	$6 + 3 + 1 = 10$ $\square + 1 = 16$ $16 - 1 = \square$ $1 + \square = 16$ $16 - \square = 1$

**Using known facts**



Children draw representations of HTO

**3+4=7**  
**leads to**  
**30+40=70**  
**leads to**  
**300+400=700**

**Bar model**



**3 + 4 = 7**

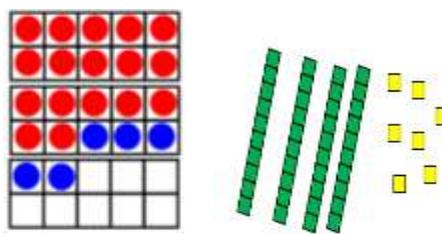


**7 + 3 = 10**

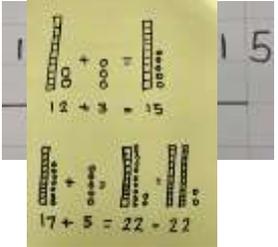
23	25
?	

**23 + 25 = 48**

**Add a two digit number and ones.**



Children explore patterns using 10's frames  
 17+5=22 / 27+5=32



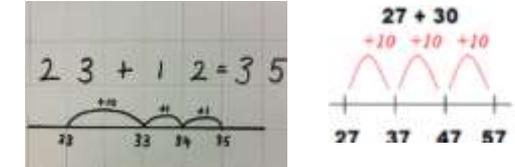
Explore related facts  
 17 + 5 = 22  
 5 + 17 = 22  
 22 - 17 = 5  
 22 - 5 = 17

**Add a 2 digit number and tens**

Use bundles of straws and Dienes to model partitioning teen numbers into tens and ones and develop understanding of place value.

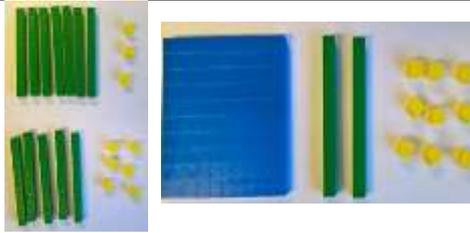


1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

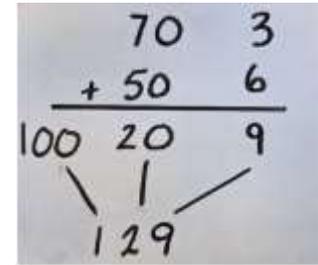
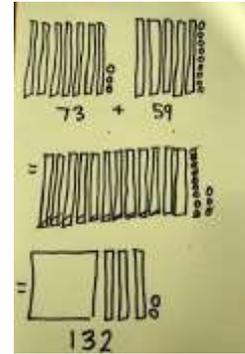


25 + 10 = 35  
 Explore that the ones digit does not change  
 27 + 10 = 37  
 27 + 20 = 47  
 27 + □ = 57

**Demonstrate an understanding of the value of ones, tens and hundreds**



Partition two-digit numbers using Base 10, e.g. 73 and 59 combined to make a new total, 132. This is made by exchanging ten '10 rods' for one '100 tile'.



### Stage Three & Four + Addition +

**VOCABULARY:** add, increase, total, plus, sum, more, altogether, column addition, estimate, inverse, double, near double, one more, ten more... one hundred more, how many more to make ...? How many more is... than ...? how much more is...?, tens boundary, hundreds boundary

**MENTAL STRATEGIES:** - Add numbers mentally, including: a three/four-digit number and a single digit number, a 3-digit number and multiples of 10, a 3-digit number and multiples of 100

Estimate the answer to a calculation and use inverse operations to check answers - Know number pairs that total 1000 (multiples of 100) - Calculate 10 or 100 more than any given number

Use knowledge of doubles to derive related facts (e.g.  $15 + 16 = 31$  because  $15 + 15 = 30$  and  $30 + 1 = 31$ )

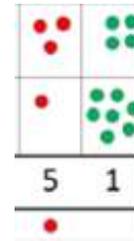
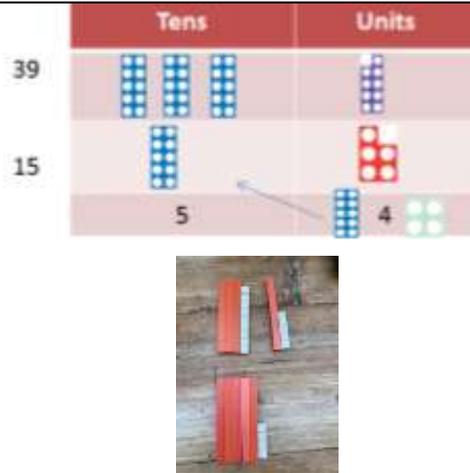
**Objective & Strategy**

**Concrete**

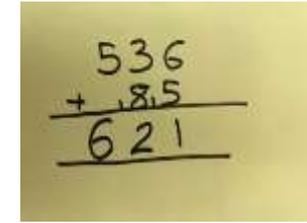
**Pictorial**

**Abstract**

**Add two or three 2 or 3 digit numbers**

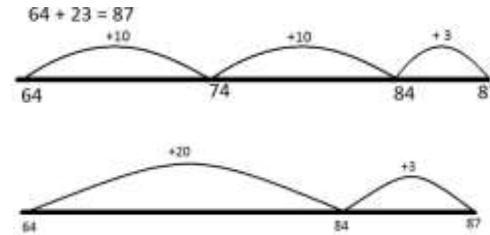


Children can draw a representation of the grid to further support their understanding, carrying the ten underneath the line.



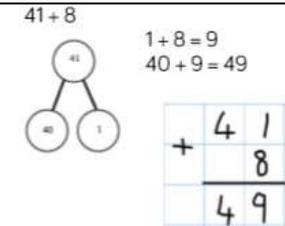
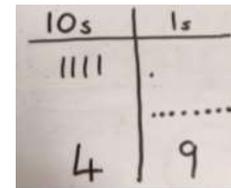
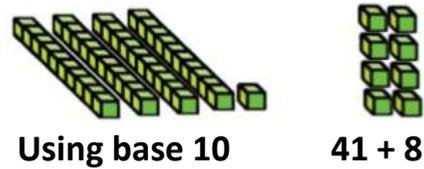
Start by partitioning the numbers before formal column to show the exchange

**And add two or three 2 or 3 digit numbers using a numberline**

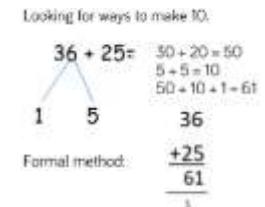
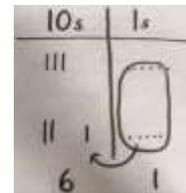
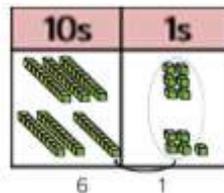


$$64 + 23 = 87$$

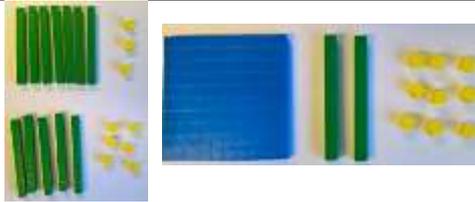
**Continue to develop understanding of partitioning and place value. TO + O**



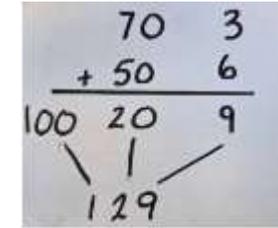
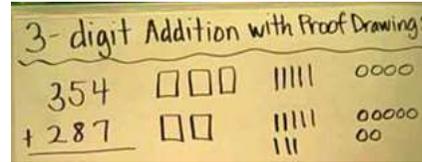
**Continue to develop understanding of partitioning and place value. TO + TO**



**Demonstrate an understanding of the value of ones, tens and hundreds**

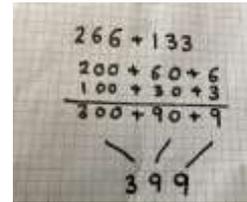


Children can partition two-digit numbers using Base 10, e.g. 73 and 59 combined to make a new total, 129. This is made by exchanging ten '10 rods' for one '100 tile'.

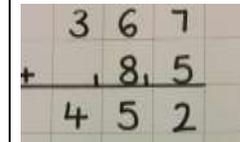
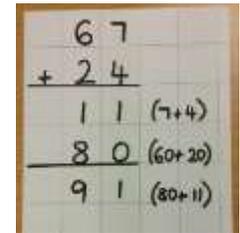


**Column addition with regrouping**

Example:  $266 + 133 = 399$



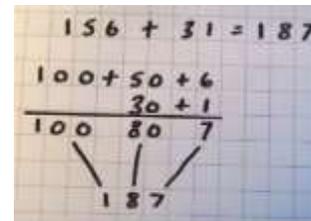
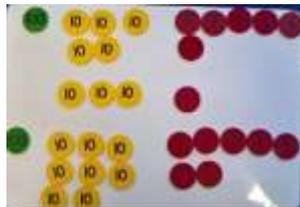
Partitioning by adding the smallest digits first (preparation for regrouping). The "extended column" method.



From this, children will begin to regroup numbers and record this accurately. The "compact column" method, (the whole calculation can be achieved in one row of calculation.)

**Column addition with no regrouping**

$156 + 31 = 187$



**Stage Five & Six + Addition +**

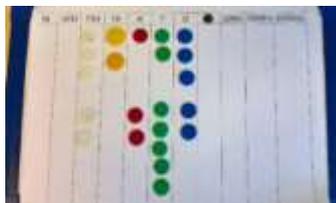
**VOCABULARY:**

Efficient written method, add, addition, column addition more, plus, increase, sum, total, altogether, score, tens boundary, hundreds boundary, thousands boundary, millions boundary, units boundary, tenths boundary, hundredths boundary, inverse, order of operations, decimal place.

**MENTAL STRATEGIES:**

- Add numbers mentally with increasingly large numbers (e.g  $10,162 + 2,300 = 12,462$ )
- Mentally add tenths (e.g  $0.2 + 0.6 = 0.8$ ) and 1-digit whole numbers and tenths ( $8 + 0.3 = 8.3$ )
- Use number bonds to 100 knowledge to calculate complements to one using hundredths (e.g  $0.83 + 0.17 = 1$ )
- Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- Add decimal numbers mentally (up to 2 decimal places)
- Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy

**Children will add numbers with more than 4-digits using the formal written method of column addition**



$$\begin{array}{r} 32123 \\ + 20252 \\ \hline 52375 \end{array}$$

$$32123 + 20252 = 52375$$

**Children will add several numbers of increasing complexity**

$$\begin{array}{r} 81059 \\ 3668 \\ 15301 \\ + 20551 \\ \hline 120579 \\ \hline \end{array}$$

$$81,059 + 3,668 + 15,301 + 20,551 = 120,579$$

**Children will add decimal numbers with the same number of decimal places using the formal written method column addition**

$$\begin{array}{r} £64.50 \\ + £19.63 \\ \hline £84.13 \end{array}$$

**Children will add several decimals numbers with a different number of decimal places**

$$\begin{array}{r} £3.59 \\ + \quad .78 \\ \hline £4.37 \end{array}$$

$$£3.59 + 78p.$$

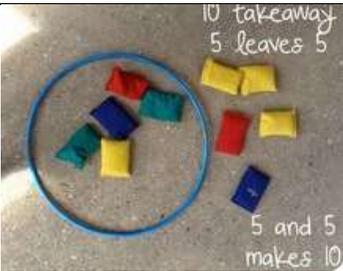
**Recognise mixed numbers and improper fractions and convert from one to the other**

$$1\frac{1}{4} = \frac{5}{4}$$

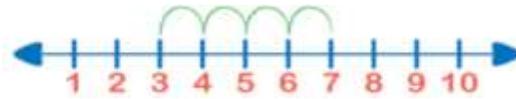
## EYFS - Subtraction -

**VOCABULARY:** take (away), leave, how many are left/left over?, how many have gone?, one less, two less... ten less..., how many fewer is... than...?, difference between, is the same as

**MENTAL STRATEGIES:** - Develop a mental image of the number system  
 Children count backwards using familiar number rhymes (e.g '10 Green Bottles', '5 Fat Sausages')  
 Count backwards from different starting points  
 Children begin to understand subtraction as taking away. Using everyday objects they start with a group, take some away and count what is left.

Objective & Strategy	Concrete	Pictorial	Abstract
Using a range of practical resources to develop their understanding of subtraction			
Listen to a subtraction stories And represent with numicon		$6 - 1 =$  Understand subtraction as 'take away'	
Children will use their fingers to help with subtraction			

Counting back  
(number lines)



**Stage One - Subtraction -**

**VOCABULARY:** number bonds, add, more, plus, make, sum, total, altogether, inverse double, near double, equals, is the same as (including equals sign), score, one more, two more... ten more, how many more to make...?, how many more is... than...?, how much more is...?

**MENTAL STRATEGIES:**

Children begin to understand subtraction as taking away. Using everyday objects they start with a group, take some away and count what is left.

Children should experience regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10. Children should memorise and reason with number bonds for numbers to 20, experiencing the = sign in different positions.

They should see addition and subtraction as related operations. E.g.  $7 + 3 = 10$  is related to  $10 - 3 = 7$ , understanding of which could be supported by an image like this. Use bundles of straws and Dienes to model partitioning teen numbers into tens and ones and develop understanding of place value. Children have opportunities to explore partitioning numbers in different ways. e.g.  $7 = 6 + 1$ ,  $7 = 5 + 2$ , Children should begin to understand addition as combining groups and counting on.

**Objective & Strategy**

**Concrete**

**Pictorial**

**Abstract**

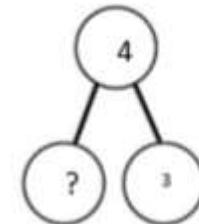
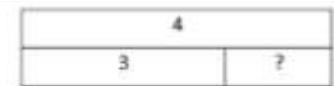
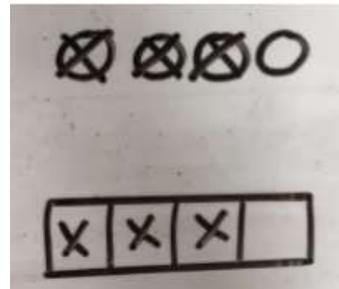
Physically taking away and removing objects from a whole (ten frames, Numicon, cubes could be used).

$4 - 3 = 1$

$7 - 2 = 5$

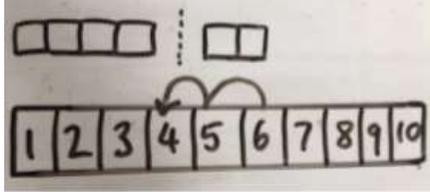
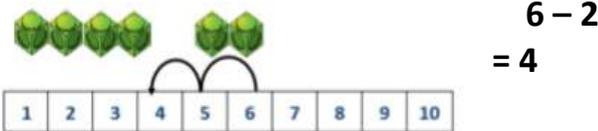
$7 - 2 = 5$

Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.

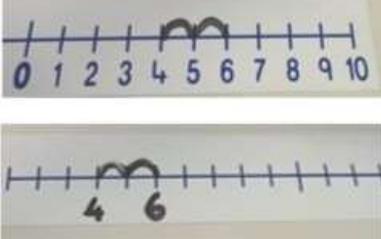


=  $4 - 3$        $4 - 3 =$

Counting back  
(using number  
lines or  
number tracks)  
children start  
with 6 and  
count back 2.



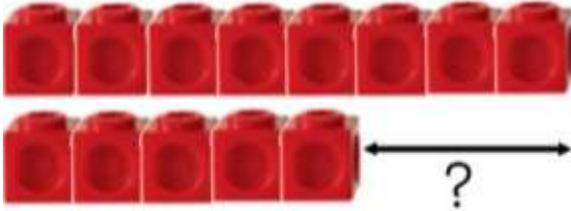
Children  
to  
represent  
what they see  
pictorially



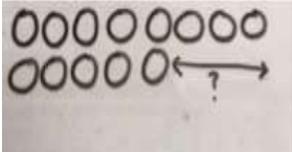
Children  
to  
represent  
the  
calculation

on a number line or number track and show their jumps.

Finding the  
difference  
(using cubes,  
Numicon or  
Cuisenaire  
rods, other  
objects can  
also be used).



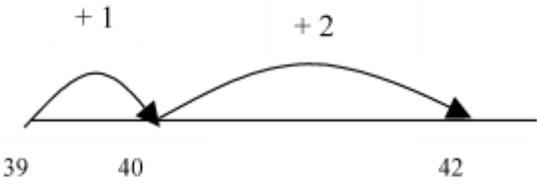
Calculate the difference between 8 and 5.



Children to draw the cubes to illustrate what they need to calculate.

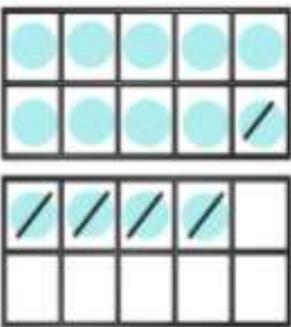
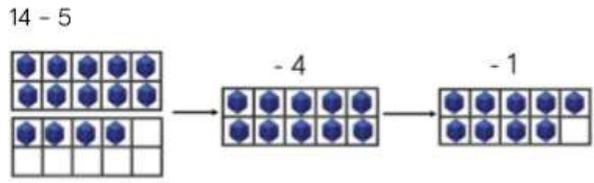
Find the difference between 8 and 5.  
8 - 5, the difference is   
Children to explore why 9 - 6 = 3  
7 - 4 have the same difference.

Find a small  
difference by  
counting up on  
a number track



$$42 - 39 = 3$$

Making 10  
using ten  
frame



$$14 - 5 = 9$$

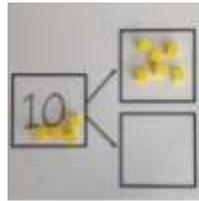
$$\begin{array}{c} 14 - 5 = 9 \\ \swarrow \quad \searrow \\ 4 \quad \quad 1 \end{array}$$

$$14 - 4 = 10$$

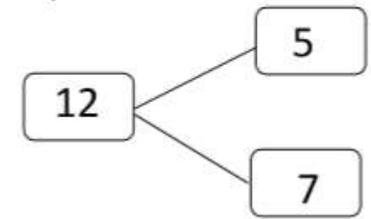
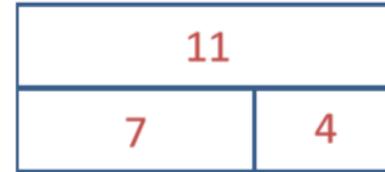
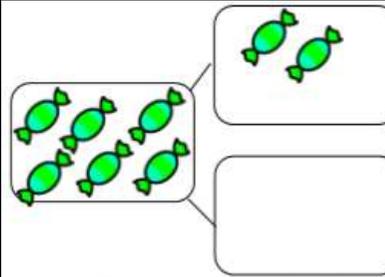
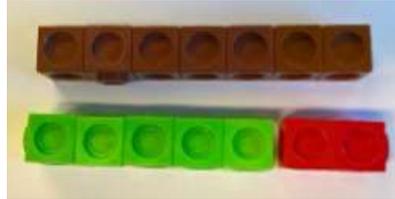
$$10 - 1 = 9$$

**Represent and use number bonds and related subtraction facts within 20**

Part Part  
Whole model



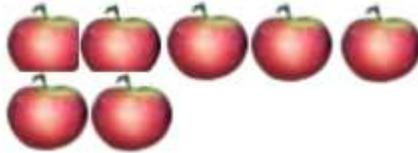
If 10 is the whole and 6 is one of the parts, what is the other part?  $10 - 6 = 4$



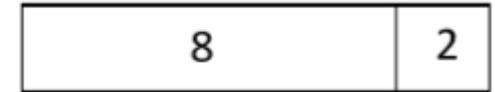
$$\begin{array}{r} 11 - \\ 4 = \\ 7 \end{array}$$

**Represent and use number bonds and related subtraction facts within 20**

Bar model

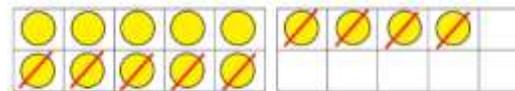
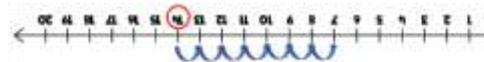
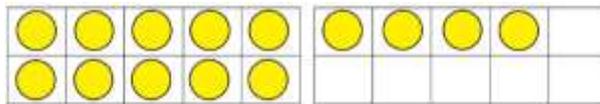


$$5 - 2 =$$



$$\begin{array}{l} 10 = 8 + 2 \\ 10 = 2 + 8 \\ 10 - 2 = 8 \\ 10 - 8 = 2 \end{array}$$

**Subtraction facts for numbers to 20 (with Ten Frames)**



$$\underline{14} - 9 = \square$$

## Stage Two - Subtraction -

### VOCABULARY:

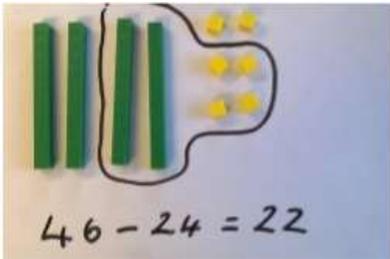
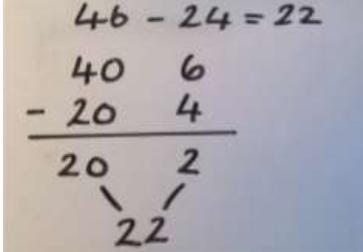
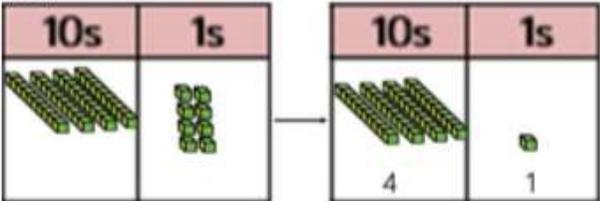
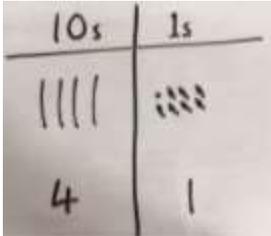
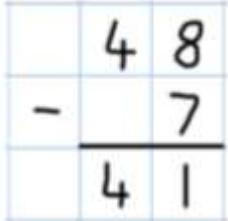
Subtract, minus, leave, how many are left/left over?, how many less is... than...?, how much fewer is...?, difference between, half, halve, equals, sign, is the same as, partition, inverse, count on , count back, one less, ten less... one hundred less.

### MENTAL STRATEGIES:

To know that subtraction is the inverse of

Use knowledge of inverse to check calculations and solve missing number problems - Subtract numbers mentally, including:  subtracting units from a 2-digit number  subtracting a multiple of 10 from a 2-digit number  subtracting a 2-digit number from another 2-digit number

Recall and use subtraction facts to 20 fluently - Use knowledge of number bonds to 100 (multiples of 10) to reason ( $40 + 60 = 100$  so  $100 - 60 = 40$  and  $100 - 40 = 60$ )

Objective & Strategy	Concrete	Pictorial	Abstract
Find the difference between 2-digit numbers (with Base 10 and expanded subtraction)	 <p style="text-align: center;"><math>46 - 24 = 22</math></p>		
Column method using base 10			



Estimate the answer to a calculation and use inverse operations to check answer

**Objective & Strategy**

**Concrete**

**Pictorial**

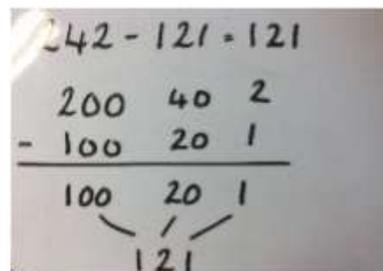
**Abstract**

**Column subtraction without regrouping (friendly numbers)**

(Base 10 – Place Value Counters)



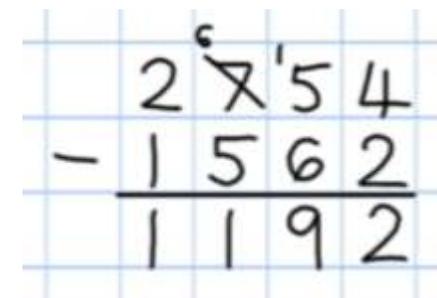
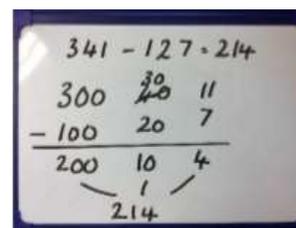
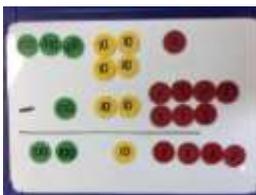
Demonstrate how to do this step by step.



$$242 - 121 = 121$$

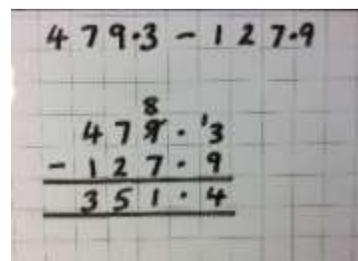
**Column subtraction with regrouping**

USE along side until children are ready for place values counters

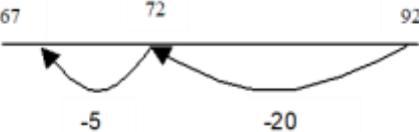


$$341 - 127 = 214$$

**Column method with regrouping (decimals- with the same amount of decimal places)**



$$479.3 - 127.9 =$$

<b>Use known number facts and place value to subtract</b>	$92 - 25 = 67$		
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### Stage Five and Six - Subtraction -

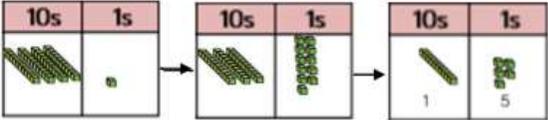
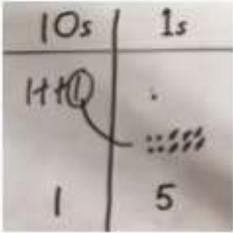
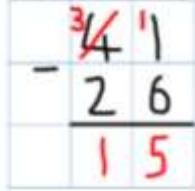
**VOCABULARY:**

Efficient written method, subtract, subtraction, minus, decrease, difference between, inverse, decimals, units , tenths and hundredths boundary, column subtraction, decomposition, exchange, order of operations.

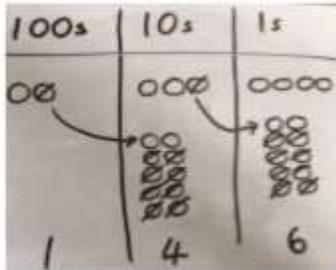
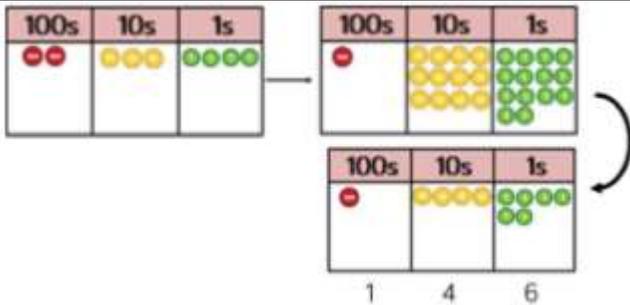
**MENTAL STRATEGIES:** Subtract increasingly large numbers mentally (e.g  $12, 654 - 1,341 = 11, 213$ ) - Mentally subtract tenths (e.g  $0.7 - 0.5 = 0.2$ ) and 1-digit whole numbers and tenths ( $8 - 0.3 = 7.7$ ) Subtract decimal numbers mentally (up to 2 decimal places)

Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy

Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.

<b>Objective &amp; Strategy</b>	<b>Concrete</b>	<b>Pictorial</b>	<b>Abstract</b>
<b>Column method using base 10 and having to exchange.</b>	Represent the base 10 pictorially, remembering to show the exchange 		Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because $41 = 30 + 11$ . 

Column method using place value counters.



Represent the place value counters pictorially; remembering to show what has been exchanged.

$$\begin{array}{r} 234 \\ - 88 \\ \hline 6 \end{array}$$

Formal column method. Children must understand what has happened when they have crossed out digits

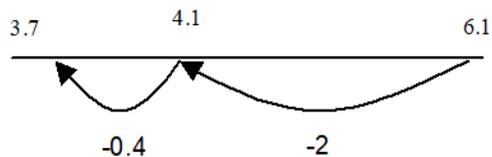
Column method with regrouping (with more than 4 digits)

$$97321 - 4964 =$$



Use known number facts and place value to subtract

$$6.1 - 2.4 = 3.7$$

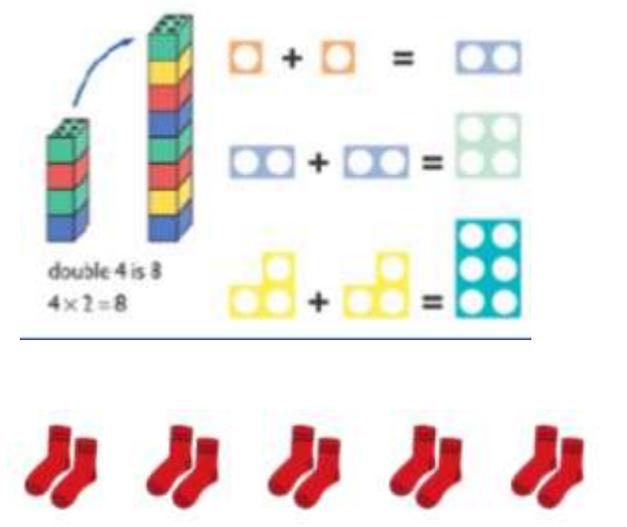
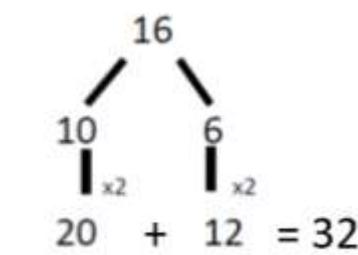


## Stage One    x Multiplication x

**VOCABULARY:** odd, even, count in twos, fives, count in tens (forwards from/backwards from), how many times? lots of, groups of, once, twice, five times, ten times , multiple of, times, multiply, multiply by, array, row, column, double.

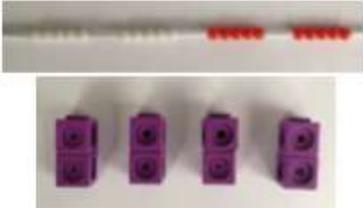
**MENTAL STRATEGIES:**

Count forwards and backwards in multiples of 2s, 5s and 10s.  
Recall doubles of numbers up to and including 10.

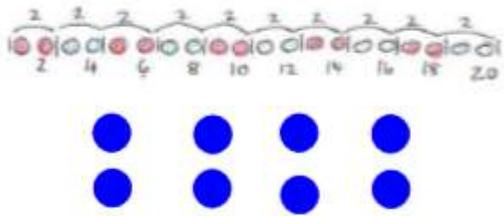
Objective & Strategy	Concrete	Pictorial	Abstract
<p><b>Doubling</b></p>	<p>Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling</p>  <p>double 4 is 8 <math>4 \times 2 = 8</math></p> <p>For example, 'double 5 is 10' because <math>5 + 5</math> is 10</p>	<p>Double 4 is 8</p> 	<p>Partition a number and then double each part before recombining it back together.</p> 

**Counting in multiples**

Count the groups as children are skip counting, children may use their fingers as they are skip counting



Children make representations to show counting in multiples.

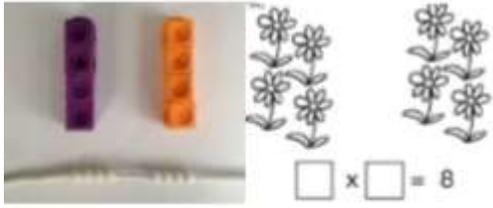


Count in multiples of a number aloud. Write sequences with multiples of numbers.

**2, 4, 6, 8, 10**

**5, 10, 15, 20, 25, 30**

**Making equal groups and counting the total**

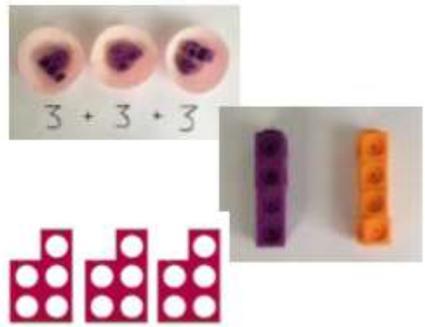


Draw  to show  $2 \times 3 = 6$

$2 \times 4 = 8$

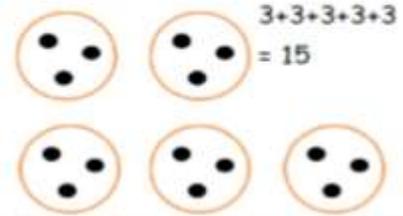
**Repeated addition**

Use different objects to add equal groups



Use different objects to add equal groups Use pictorial including number lines to solve prob

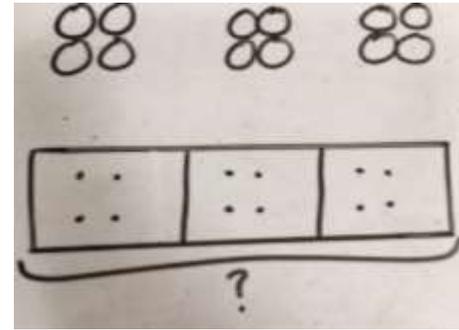
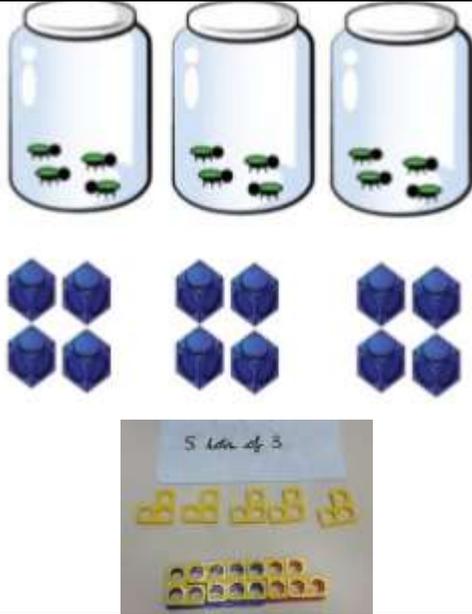
There are 3 sweets in one bag. How many sweets are in 5 bags altogether?



Write addition sentences to describe objects and pictures



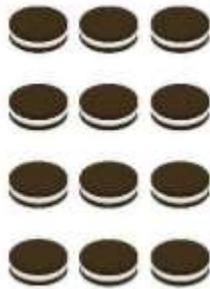
**Repeated grouping/repeated addition**  $3 \times 4$   $4 + 4 + 4$  There are 3 equal groups, with 4 in each group.



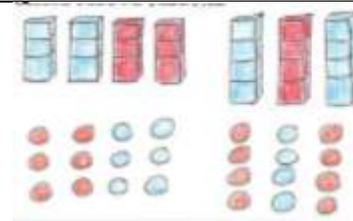
$3 \times 4 = 12$   
 $4 + 4 + 4 = 12$

**Understanding arrays**

Children begin to organise their multiplication calculations more efficiently using arrays:

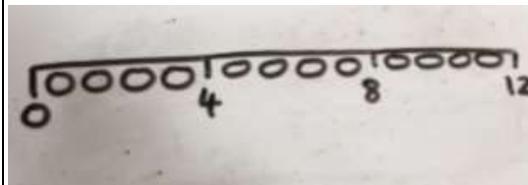
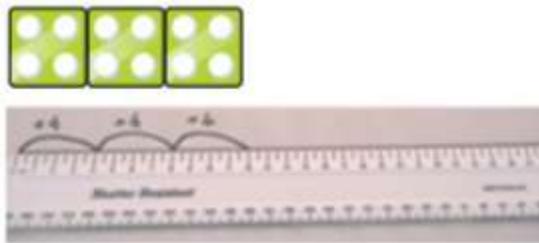


E.g. The cookies are organised into 4 rows of 3. There are 3 cookies in each row (or set = factor). There are 4 rows in total (number of sets = factor). There are 12 cookies altogether (the total = product). We can write this as 3 cookies  $\times$  4 = 12 cookies

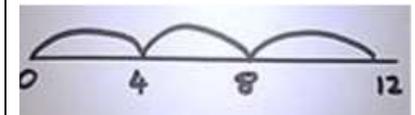


$3 \times 2 = 6$   
 $2 \times 5 = 10$

**Number lines to show repeated groups-  $3 \times 4$**



**Abstract number line showing three jumps of four.**  
 $3 \times 4 = 12$

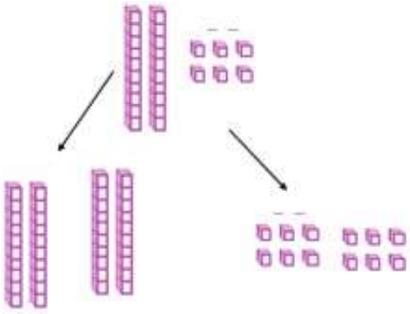
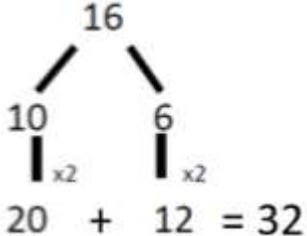


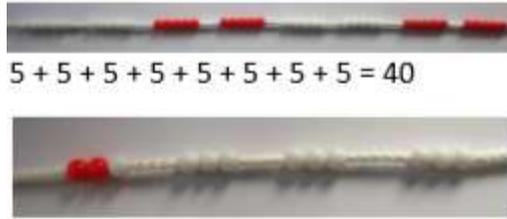
## Stage Two    x Multiplication x

**VOCABULARY:** odd, even, twos, fives, tens, threes, lots of, groups of, once, twice, three times, five times, ten times, multiple of, times, multiply, multiply by, repeated addition, array, row, column, double.

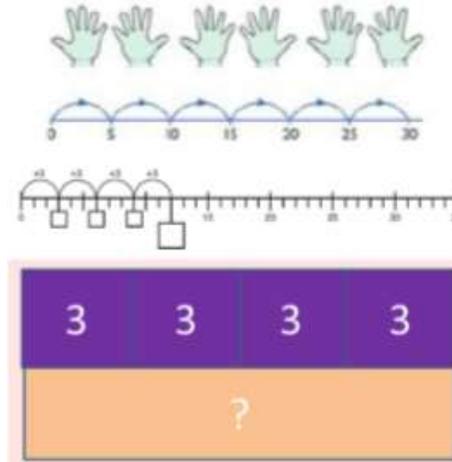
**MENTAL STRATEGIES:**

Count forwards and backwards in multiples of 3.  
 Know the 2, 5 and 10 times tables (in and out of order)  
 Recognise odd and even numbers

Objective & Strategy	Concrete	Pictorial	Abstract
<p><b>Doubling</b></p>	<p>Model doubling using dienes and PV counters</p>  <p style="text-align: center;"><math>40 + 12 = 52</math></p>	<p>Draw pictures and representations to show how to double numbers</p>	<p>Partition a number and then double each part before recombining it back together.</p>  <p style="text-align: center;"><math>20 + 12 = 32</math></p>
<p><b>Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)</b></p>	 <p style="text-align: center;"><math>10 \times 1</math>      <math>10 \times 2</math>      ...      <math>10 \times 5</math></p> <p>Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.</p>	<p>Number lines, counting sticks and bar models</p>	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>0, 2, 4, 6, 8, 10              0, 3, 6, 9, 12, 15              0, 5, 10, 15, 20, 25, 30</p> <p style="text-align: center;"><math>4 \times 3 = \square</math></p>

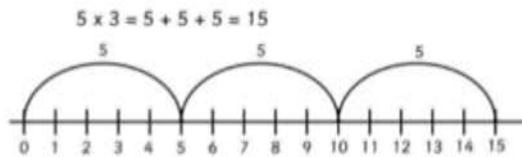


should be used to show representation of counting in multiples.

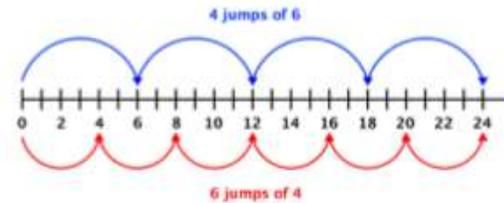


**Repeated addition (with Cuisenaire rods and number lines)**

The number line can help children to see the effect of multiplying the number 5 is the same as adding 5 each time. So  $5 \times 3$  is the same as  $5 + 5 + 5 = 15$ .



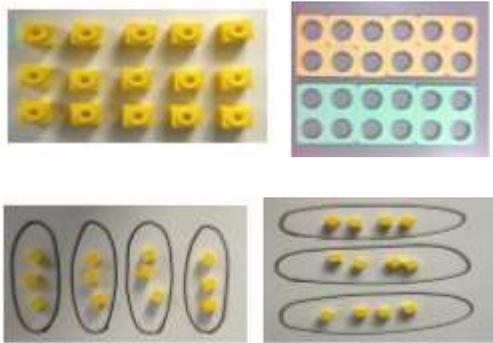
Children can represent the concept of commutativity using a number line.



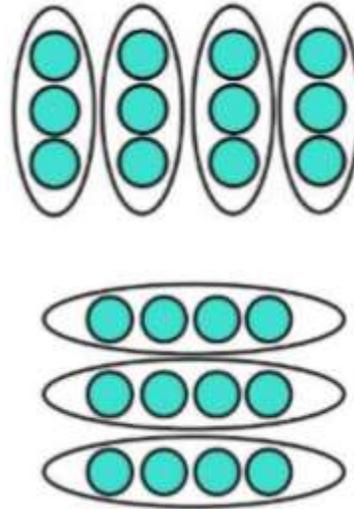
In this example  $6 \times 4$  (blue) =  $4 \times 6$  (red) and it is also true that  $6 + 6 + 6 + 6 = 4 + 4 + 4 + 4 + 4 + 4$

**Multiplication is commutative**

Create arrays using counters and cubes and Numicon. Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer



Use representations of arrays to show different calculations and explore commutativity.



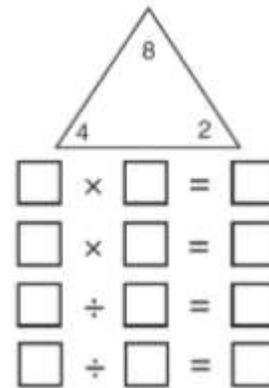
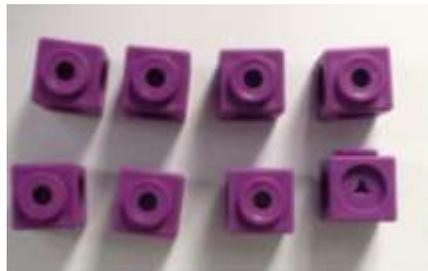
**$12 = 3 \times 4$**   
 **$12 = 4 \times 3$**

Use an array to write multiplication sentences and reinforce repeated addition.



$5 + 5 + 5 = 15$   
 $3 + 3 + 3 + 3 + 3 = 15$   
 $5 \times 3 = 15$   
 $3 \times 5 = 15$

**Using the Inverse**  
 This should be taught alongside division, so pupils learn how they work alongside each other



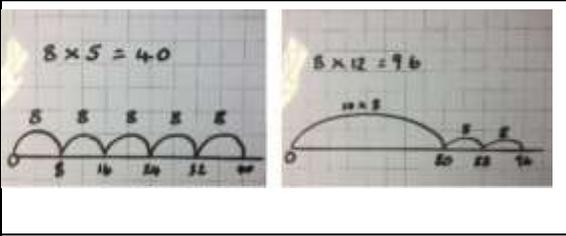
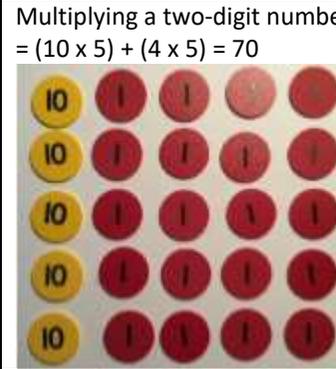
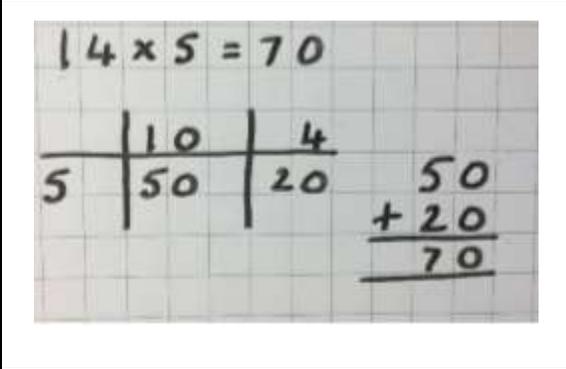
**$2 \times 4 = 8$**   
 **$4 \times 2 = 8$**   
 **$8 \div 2 = 4$**   
 **$8 \div 4 = 2$**   
 **$8 = 2 \times 4$**   
 **$8 = 4 \times 2$**   
 **$2 = 8 \div 4$**   
 **$4 = 8 \div 2$**   
**Show all 8 related fact family sentences**

### Stage Three    x Multiplication x

**VOCABULARY:** multiply, times, groups of, equal groups of, multiple of, multiplied by, estimate, inverse, grid multiplication, expanded column multiplication, partition, commutative, associative, product.

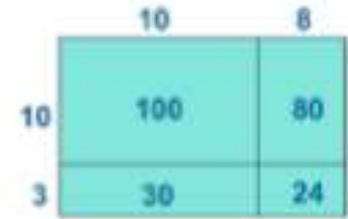
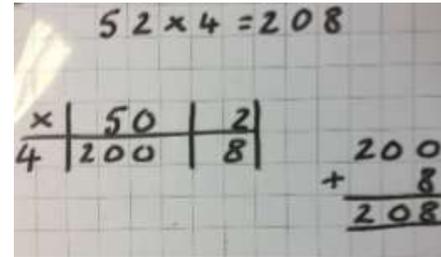
**MENTAL STRATEGIES:**

Count forwards and backwards in multiples of 4, 8, 50 & 100 - Know the 3, 4 and 8 times tables (in and out of order) - Connect the 2, 4 and 8 times tables through doubling - Use knowledge of place value to calculate multiplication (e.g.  $2 \times 2 = 4$ ,  $2 \times 20 = 40$ ,  $2 \times 200 = 400$ )

Objective & Strategy	Concrete	Pictorial	Abstract						
<b>Repeated addition (using a number line)</b>	Progressing from multiplying by one digit (for example: $8 \times 5 = 40$ ) to multiplying by two digits (e.g. $8 \times 12$ )								
<b>Arrays and grid method for partitioning to multiply (place value counters)</b>	Multiplying a two-digit number by one digit, for example: $14 \times 5 = (10 \times 5) + (4 \times 5) = 70$ 		<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td> <td>30</td> <td>5</td> </tr> <tr> <td>7</td> <td>210</td> <td>35</td> </tr> </table> <p style="text-align: center;"><math>210 + 35 = 245</math></p>	x	30	5	7	210	35
x	30	5							
7	210	35							

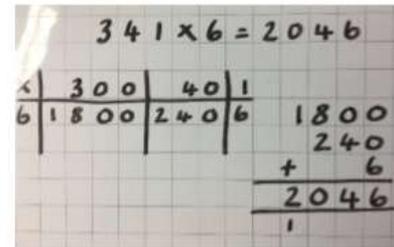
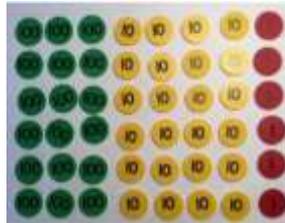
**Multiplying a larger two-digit number by one digit**

for example:  $52 \times 4 = (50 \times 4) + (2 \times 4) = 208$



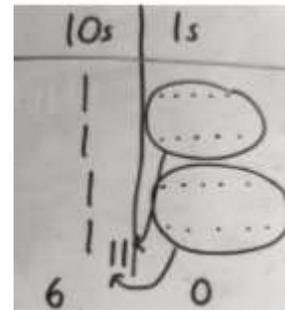
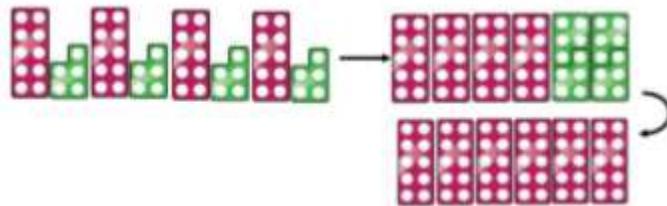
**Progressing to multiplying a three-digit number by a one-digit number**

for example:  $341 \times 6 = 2046$



**Partition to multiply using Numicon, base 10 or Cuisenaire rods**

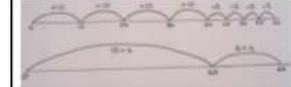
$4 \times 15$



Children to be encouraged to show the steps they have taken.

$$\begin{array}{r}
 4 \times 15 \\
 \swarrow \searrow \\
 30 \quad 5 \\
 30 \times 4 = 40 \\
 5 \times 4 = 20 \\
 40 + 20 = 60
 \end{array}$$

A number line can also be used

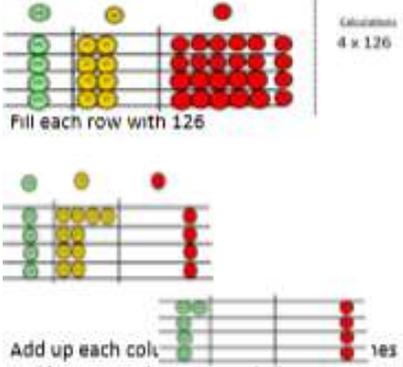
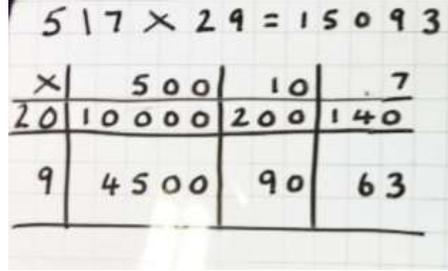
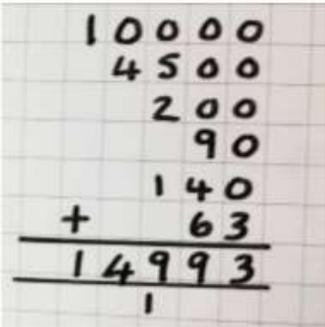
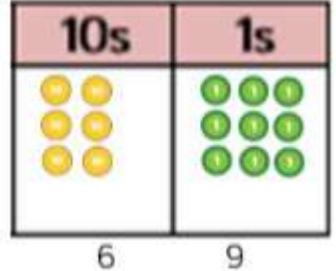
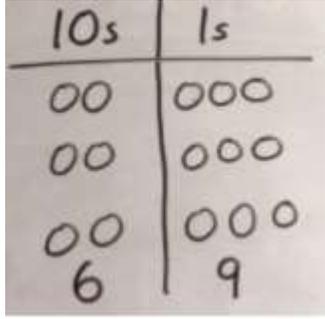


**VOCABULARY:**

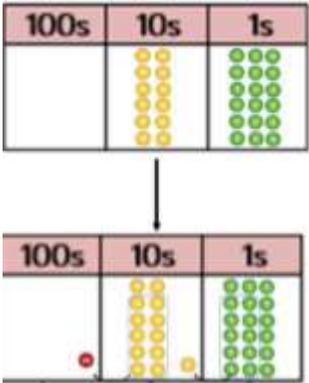
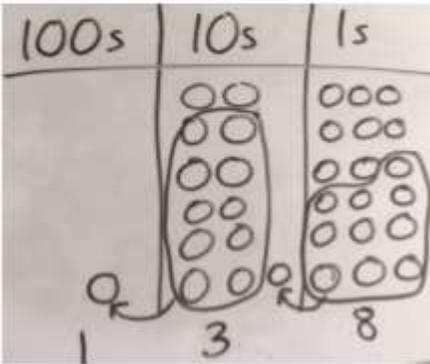
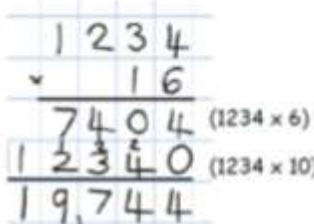
Multiply, multiplied by, product, short multiplication, partition, distributive law, commutative, groups of, multiply, times, multiples, inverse.

**MENTAL STRATEGIES:**

Know all times tables up to and including 12 x 12 (by the end of Year 4) - Recognise and use factor pairs (e.g factor pairs for numbers up to and including 10) - Know that TU x 5 is TU x 10 then divide by 2 (e.g  $18 \times 5 = (18 \times 10) \div 2 = 90$ ) - Know that TU x 9 is TU x 10 then subtract TU (e.g  $18 \times 9 = (18 \times 10) - 18 = 162$ )

Objective & Strategy	Concrete	Pictorial	Abstract
<p><b>Grid method for multiplying larger numbers</b></p>	<p>Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 row</p>  <p>Fill each row with 126</p> <p>Add up each col. 100s 10s 1s</p>	 <p><math>517 \times 29 = 15093</math></p>	
<p><b>Formal column method with place value counters (base 10 can also be used.)</b></p>	<p><math>3 \times 23</math></p> 	<p>Children to represent the counters pictorially.</p> 	<p>Children to record what it is they are doing to show understanding.</p>

			$  \begin{array}{r}  3 \times 23 \\  \swarrow \searrow \\  20 \quad 3  \end{array}  $ $  \begin{array}{r}  3 \times 20 = 60 \\  3 \times 3 = 9 \\  60 + 9 = 69  \end{array}  $ $  \begin{array}{r}  23 \\  \times 3 \\  \hline  69  \end{array}  $
	Watch utube		
<b>Stage Five    x Multiplication x</b>			
<b>VOCABULARY:</b> composite numbers, prime number, prime factor, cube number, square number, derive, factor pairs, formal written method, times, multiply, multiplied by, multiple of, product, short multiplication, partition, long multiplication, scaling, decimal place, units, tenths and hundreds.			
<b>MENTAL STRATEGIES:</b> Recognise and calculate factor pairs for any number - Use times table knowledge to derive multiples of any number - Establish whether a number is a prime number (up to 100) or a composite number (not prime) and recall prime numbers up to 19 - To know what a square number is and recall all square numbers (up to and including 144) - To know what a cube number is and recall the first 5 cube numbers			
<b>Objective &amp; Strategy</b>	<b>Concrete</b>	<b>Pictorial</b>	<b>Abstract</b>

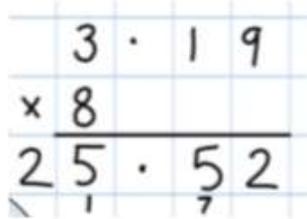
<p><b>Formal column method with place value counters</b></p>	<p>6 x 23</p> 		$6 \times 23 =$ $\begin{array}{r} 23 \\ \times 6 \\ \hline 138 \\ \hline 11 \end{array}$
<p><b>Column multiplication</b></p>	<p>Manipulatives may still be used with the corresponding long multiplication modelled alongside.</p>		 <p>18 x 3 on the first row (8 x 3 = 24, carrying the 2 for 20, then 1 x 3)</p> <p>18 x 10 on the 2nd row. Show multiplying by 10 by putting zero in units first</p> 
<p><b>Use the grid method to solve multiplication calculations.</b></p>	<p>First multiplication calculations that are a two digit number multiplied by a one digit number. Next a two digit number by a two digit number. Followed by extending the numbers beyond 3 digits</p>		

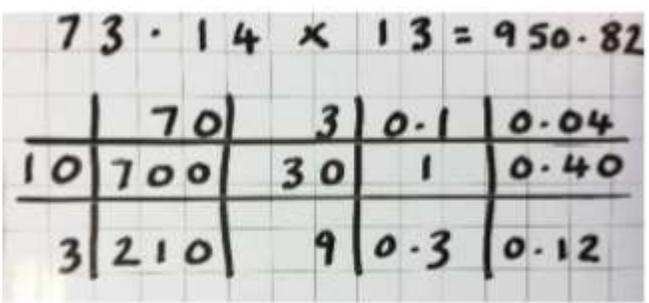
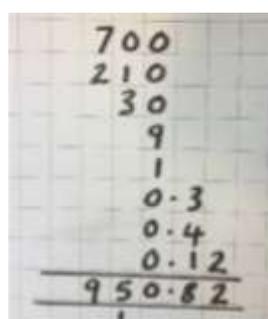
	where one of the numbers might have a decimal, using place value knowledge to check that digits show the correct value in their answer.		
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**Stage Six    x Multiplication x**

**VOCABULARY:** common factors, multiples, prime, formal written method, multiply, multiplied by, multiple of, product, short and long multiplication, partition, scaling, decimal place, units, tenths and hundredths.

**MENTAL STRATEGIES:** Use scaling to solve decimal number problems as whole number problems using the rule: ‘the number of decimal digits in the question is the same as the number of decimal digits in the answer’ - Identify common factors, common multiples and prime numbers - Use common factors to simplify fractions mentally - Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.

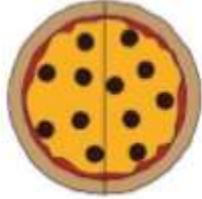
<b>Objective &amp; Strategy</b>	<b>Concrete</b>	<b>Pictorial</b>	<b>Abstract</b>
<b>Multiplying decimals up to 2 decimal places by a single digit</b>	<b>73.14 x 13</b>		Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer.  

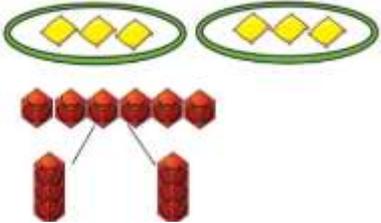
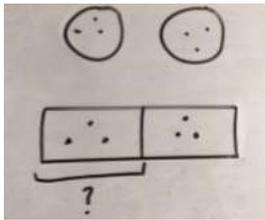
<p><b>Grid method for multiplying decimal numbers (with up to 2 d.p.)</b></p>			
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**Stage One      Division**

**VOCABULARY:** halve, share, share equally, groups, equal groups of, divide, divided by, left, left over

**MENTAL STRATEGIES:**  
 Develop a mental image of the number system.  
 Understand the value of a number  
 Count forwards and backwards in multiples of 2s, 5s and 10s.

<b>Objective &amp; Strategy</b>	<b>Concrete</b>	<b>Pictorial</b>	<b>Abstract</b>
<p><b>Children will understand equal groups and share objects into groups in play scenarios</b></p>			
<p><b>Children will be taught to associate 'half' with dividing by two and recognise, find and name a half as one of two equal parts</b></p>	<p>Can you cut the pizza in half?</p>		

<p><b>Sharing using a range of objects</b></p>			<p><math>6 \div 2 = 3</math></p> <table border="1" data-bbox="1697 169 2040 225"> <tr> <td>3</td> <td>3</td> </tr> </table> <p>Children should also be encouraged to use their 2 times tables facts.</p>	3	3
3	3				

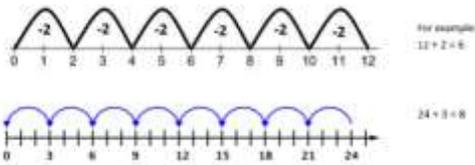
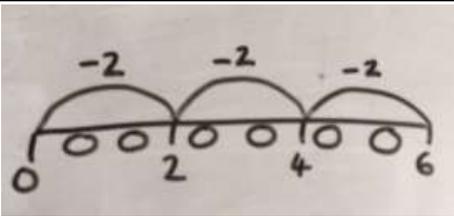
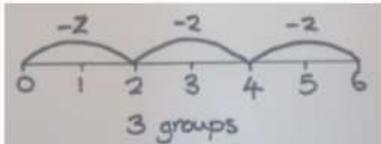
<p><b>Children will recognise and write the division symbol (<math>\div</math>) in mathematical statements, calculating the answer with the teacher using concrete objects</b></p>			<p><math>8 \div 2 = 4</math></p>
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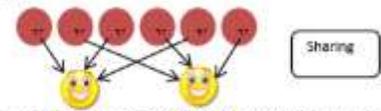
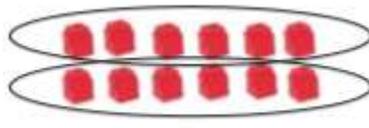
## Stage Two    Division

**VOCABULARY:** groups of, equal groups of, halve, share, share equally, divide, divided by, divided into, repeated subtraction, inverse.

**MENTAL STRATEGIES:**

- To know that division is the inverse of multiplication    - Recall division facts for the 2, 5 and 10 times tables
- Recall halves for even numbers up to and including 20

Objective & Strategy	Concrete	Pictorial	Abstract
<p><b>Children will understand the operation of division as grouping using repeated subtraction on a prepared number line</b></p>			<p>Abstract number line to represent the equal groups that have been subtracted.</p> 

<p><b>Children will be able to represent a division calculation using an array and write the division within a number sentence</b></p>	<p>In these questions, the children have divided 24 between 8 people.</p> 		$24 \div 8 = 3$
<p><b>Children will be taught to understand the difference between sharing and grouping. Children will also connect unit fractions to equal sharing and grouping</b></p>		<p>If 6 sweets are shared between 2 people, how many do they get each?</p>  <p>If there are 5 sweets, how many people can have 2 sweets each?</p> 	
<p><b>Children will solve one-step division problems (including missing number problems) using concrete objects and pictorial representations</b></p>			$12 \div \blacksquare = 6$

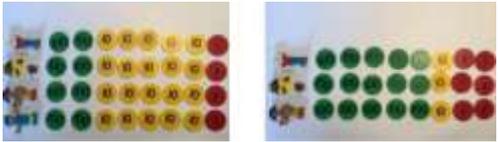
### Stage Three      Division

**VOCABULARY:** divided by, divide, divided into, grouping, divisor, short division, remainder, inverse.

**MENTAL STRATEGIES:**

- Know the division facts from the 3, 4 and 8 times tables
- Use knowledge of place value to calculate division (e.g.  $14 \div 2 = 7$ ,  $140 \div 2 = 70$ ,  $1400 \div 2 = 700$ )

<b>Objective &amp; Strategy</b>	<b>Concrete</b>	<b>Pictorial</b>	<b>Abstract</b>
<p><b>Children will use practical resources to support the Division within arrays (TU ÷ U) (place value counters are used to create an array)</b></p>			<p>In this example, 466 divided by 2, the counters can be easily divided into 2 equal rows without any regrouping or exchanging. <math>466 \div 2 = 233</math> Each 'person' gets 233</p>

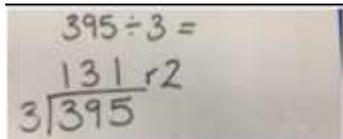
<p>Children will use practical resources to support the short division method and will be encouraged to use multiples of the divisor to assist (HTU ÷ U)</p>			<p>In these examples, <math>1008 \div 4 = 251</math> and <math>1536 \div 3</math>, children will start with a '1000 counter' and exchange it for ten '10 counters' to make the sharing easier. If there are any '100s' left over, these are exchanged for '10s' and so on until the answer is reached. If any counters are left over at the end these are the remainders.</p>
<p>Children will use practical resources to support solving division number sentences with remainders (TU ÷ U)</p>			<p><math>693 \div 3</math> In this example, there are two counters left over so the answer is 231 r2</p>

### Stage Four      Division

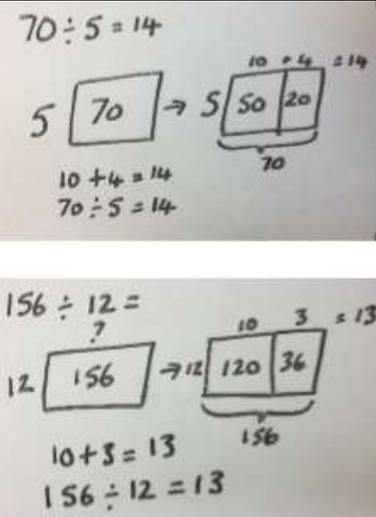
**VOCABULARY:** factor, divisor, divided by, divided into, remainders, divisible by, equivalent, short division, derive, Quotient, inverse, remainder, multiples, exchange.

**MENTAL STRATEGIES:**

Know all related division facts for all times tables up to 12 times table

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Children will use practical resources to support solving division number sentences with remainders (HTU ÷ U)</p>			



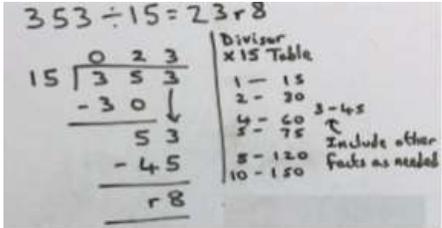
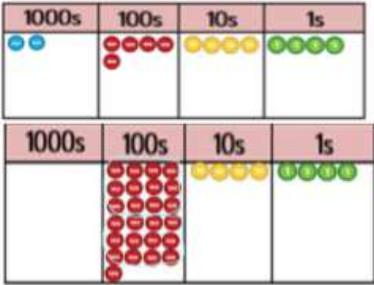
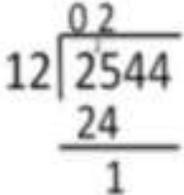
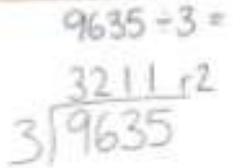
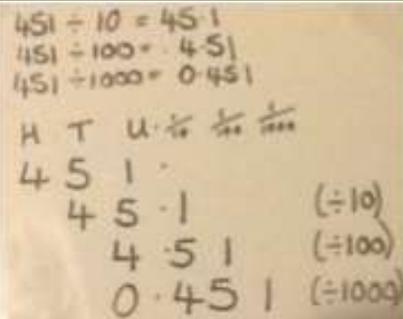
<p><b>Division within 'open arrays' (pictorial arrays)</b></p>	<p>In the following examples, children can find the answer by using known multiplication facts. In this example, <math>70 \div 5</math>, 70 has been partitioned into two other numbers <math>50 + 20</math> because both 50 and 20 are multiples of 5. <math>70 \div 5 = (50 \div 5) + (20 \div 5)</math> <math>70 \div 5 = 10 + 4</math></p>		
<p><b>Expanded division method (also referred to as 'chunking')</b></p>	<p>This method is similar to the 'open array' method, because multiplication facts are used for dividing. In this example, <math>172 \div 8</math>, multiples of 8 are subtracted from 172. If, after subtracting multiples of 8, there is an amount left over then that is the remainder. The number of 8's that are subtracted are recorded in brackets and then added together to find the answer.</p>		$172 \div 8 = 21r4$ $\begin{array}{r} 172 \\ -80 \quad (10 \times 8) \\ \hline 92 \\ -80 \quad (10 \times 8) \\ \hline 12 \\ -8 \quad (1 \times 8) \\ \hline 4 \quad r4 \end{array}$

### Stage Five    Division

**VOCABULARY:** divide, divided by, divided into, divisible by, remainder, quotient, inverse, decomposing, factor, decimal place, units, tenths, scaling, short division

**MENTAL STRATEGIES:**

- Multiply and divide numbers mentally drawing upon known facts
- Associate fractions with division

Objective & Strategy	Concrete	Pictorial	Abstract
Children will use short division to solve division number sentences with remainders (HTU ÷ TU)			
Children will use practical resources to support solving division number sentences with remainders (HTU ÷ U)			
Children will use practical resources to support solving division number sentences with remainders (THTU ÷ U)			
Children will learn to divide whole numbers and those involving decimals by 10, 100 and 1000 by moving the digits around the fixed decimal			

### Stage Six Division

**VOCABULARY:** divide, divided by, divided into, divisible by, remainder, factor, quotient, inverse, decimal place, units, tenths,

hundredths, scaling, formal written methods.

**MENTAL STRATEGIES:**

- Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy - Calculate a fraction of an amount

Objective & Strategy	Concrete	Pictorial	Abstract
<p><b>Short division (up to 4 digits by a 1 digit number interpret remainders appropriately for the context)</b></p>		<p><b>Short division</b></p> <p>98 ÷ 7 becomes</p> $\begin{array}{r} 14 \\ 7 \overline{) 98} \end{array}$ <p>Answer: 14</p> <p>432 ÷ 5 becomes</p> $\begin{array}{r} 86 \text{ r } 2 \\ 5 \overline{) 432} \end{array}$ <p>Answer: 86 remainder 2</p> <p>496 ÷ 11 becomes</p> $\begin{array}{r} 45 \text{ r } 1 \\ 11 \overline{) 496} \end{array}$ <p>Answer: 45 <math>\frac{1}{11}</math></p>	
<p><b>Long division (interpret remainders as whole numbers, fractions or round)</b></p>		<p>We can't group 2 thousands into groups of 12 so will exchange them.</p> <p>We can group 24 hundreds into groups of 12 which leaves with 1 hundred.</p> <p>After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 te</p> <p>After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.</p>	$\begin{array}{r} 02 \\ 12 \overline{) 2544} \\ \underline{24} \\ 1 \\ 021 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2 \end{array}$ $\begin{array}{r} 0212 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$

**Divide numbers decimal  
numbers with up to 3  
decimal places by 10, 100  
and 1000 by moving the  
digits around a fixed  
decimal**

Handwritten mathematical examples showing decimal division by 10, 100, and 1000, along with a place value chart.

$$\begin{aligned} 31.2 \div 10 &= 3.12 \\ 31.2 \div 100 &= 0.312 \\ 31.2 \div 1000 &= 0.0312 \end{aligned}$$

Place value chart:

H	T	U	T	H	U	T	H	U	T
3	1	.	2						

Results shown to the right of the chart:

- 3.12 ( $\div 10$ )
- 0.312 ( $\div 100$ )
- 0.0312 ( $\div 1000$ )